IX. *On the existence of an element of Strength in Beams subjected to Transverse Strain, arising from the Lateral Action of the fibres or particles on each other, and named by the author the 'Resistance of Flexure.'* *By* William Henry Barlow, *Esq., F.R.S.*

Received February 23,—Read March 29, 1855.

It has been long known, that under the existing theory of beams, which recognizes only two elements of strength, namely, the resistances to direct compression and extension, the strength of a bar of cast iron subjected to transverse strain cannot be reconciled with the results obtained from experiments on direct tension, if the neutral axis is in the centre of the bar.

The experiments made both on the transverse and on the direct tensile strength of this material have been so numerous and so carefully conducted, as to admit of no doubt of their accuracy; and it results from them, either that the neutral axis must be at, or above, the top of the beam, or there must be some other cause for the strength exhibited by the beam when subjected to transverse strain.

In entering upon this question, it became necessary to establish clearly the position of the neutral axis, and the following experiments were commenced with that object; but they have led to others, which are also described herein, and which establish the existence of a third, and a very important element of strength in beams.

I was desirous that the experiments for determining the position of the neutral axis should be made on such a scale and in such a manner as to place this question beyond doubt; and with this object the following means were adopted:—

Two beams were cast, 7 feet long, 6 inches deep, and 2 inches in thickness; on each of which were cast small vertical ribs at intervals of 12 inches: these ribs were one-fourth of an inch wide, and projected one-fourth of an inch from the beam. In each rib nine small holes were drilled to the depth of the surface of the beam, for the purpose of inserting pins attached to a delicate measuring instrument; the intention being to ascertain the position of the neutral axis by measuring the distance of the holes in the vertical ribs when the beam was placed under different strains. The measuring instrument consisted of a bar of box-wood, in which was firmly inserted, at one end, a piece of brass, carrying a steel pin; and at the other end a similar piece of brass carrying the socket of an adjusting screw. The adjusting screw moved a brass slide, in the manner shown in Plate XII. which carried another pin similar to that inserted in the box-wood bar, at the other end of the instrument. The instrument was first made entirely of brass; but the effects of expansion from the heat of the hand were so sensible, that the wooden bar was substituted. The pins on the instrument fitted loosely into holes in the beam; and the mode of using the instrument was, to bring the pins up by means of the screw against the side of the holes with a certain degree of pressure, which, with a little practice in using the instrument, was attained with considerable accuracy.

Two beams were employed in order to avoid errors which might arise from accidental irregularities in the metal. The head of the adjusting screw was graduated to 100 divisions, and the screw had 43·9 threads to the inch, so that one division was equal to $\frac{1}{4390}$th of an inch.

The measurements were, in all cases, taken by the outsides of the pins of the measuring instrument; and when the instrument read zero, the actual distance of the outer sides of the two pins was $\frac{51661}{4390}$ inches, so that the constant number 51661 being added to the micrometer readings gives, in each case, the total distance in terms of $\frac{1}{4390}$th of an inch. The form and dimensions of these beams are given in Plate XIII.

The measurements were taken four times in each position of the beam, and the error of measurement did not generally exceed from one to two divisions; but if in the four observations an error amounting to more than four was found, it was corrected by remeasurement.

The numbers given in the following Tables are the micrometer readings, and the *means* of four observations in each case. In these experiments more than 3000 measurements were taken; but to avoid unnecessary figures, only the more prominent results are given.

Table No. I. contains the measurements of the centre division of the first beam under eight different conditions.

Table No. II. contains similar measurements of the second beam.

In the first experiment it was found that, when the beam was inverted, the measuring instrument appeared to bear upon a different part of the holes, so that a direct comparison between the distances, in the beam erect and inverted, cannot be made with the same accuracy as the comparisons of different strains upon the beam when in the same position. The first beam had been subjected to strain for the purpose of testing the measuring instrument previous to these experiments being made; but the second beam had not; and it will be seen that the effect of the strains in the latter case caused a permanent lengthening of the beam. The same strain was frequently applied afterwards, but I could not observe any increase of this effect. There was certainly a further apparent lengthening of both beams; but I ascertained that this arose from a slight wearing of the working parts of the measuring instrument, from the great number of measurements taken. In both experiments the beam was measured, first, in an erect position; and secondly, inverted; but in the Tables, the measurements of the same parts of the beam are placed opposite each other, so that they may be compared throughout with greater facility.

Determination of the Neutral Axis.

Measurements of the First Beam.

Beam erect.					Beam inverted.									
No. 1.		No. 2.		No. 3.	No. 4.		No. 5.		No. 6.		No. 7.		No. 8.
At rest previous to being strained.	Difference.	Strained with 7373 lbs. on the end, equal to 14,746 lbs. on the centre.	Difference.	Weight taken off, condition the same as No. 1.	Beam reversed, bearing its own weight on the centre.	Difference.	Strain of 2893 lbs. on the end, equal to 5786 lbs. on the centre.	Difference.	Strain of 5133 lbs. on the end, equal to 10,066 lbs. on the centre.	Difference.	Strain of 7373 lbs. on the end, equal to 14,706 lbs. on the centre.	Difference.	Weight taken off, condition the same as No. 4.
Micrometer readings. 2208	+70	Micrometer readings. 2278	−67	Micrometer readings. 2211	Micrometer readings. 2210	−33	Micrometer readings. 2177	−25	Micrometer readings. 2152	−26	Micrometer readings. 2126	+71	Micrometer readings. 2197
2186	+55	2241	−53	2188	2187	−25	2162	−16	2146	−21	2125	+53	2178
2095	+36	2131	−33	2098	2103	−20	2083	−12	2071	−13	2058	+36	2094
2127	+14	2141	−13	2128	2129	−10	2119	− 8	2111	− 5	2106	+21	2127
2110	− 5	2105	+ 5	2110	2117	− 2	2115	− 1	2114	+ 2	2116	——	2116
2052	−21	2031	+13	2054	2060	+ 5	2065	+ 6	2071	+12	2083	−20	2063
2095	−39	2056	+42	2098	2101	+15	2116	+14	2130	+19	2149	−37	2112
2052	−58	1994	+68	2052	2056	+21	2077	+21	2098	+29	2127	−60	2067
2101	−73	2028	+76	2104	2111	+28	2139	+26	2165	+34	2199	−73	2126

Note.—The extensions are marked + ; the compressions are marked −.

Determination of the Neutral Axis.

Measurements of the Second Beam.

Beam erect.								Beam inverted.				
No. 1. At rest previous to being strained.	Difference.	No. 2. Strain of 8000 lbs. on centre.	Difference.	No. 3. Strain of 16,000 lbs. on centre.	Difference.	No. 4. Weight removed.	Difference.	No. 5. Strain of 8000 lbs. on centre.	Difference.	No. 6. Strain of 16,000 lbs. on centre.	Difference.	No. 7. Weight removed.
Micrometer readings.		Micrometer readings.		Micrometer readings.		Micrometer readings.		Micrometer readings.		Micrometer readings.		Micrometer readings.
1633	+37	1670	+65	1735	−89	1646	−44	1602	−56	1546	+97	1633
1525	+28	1553	+47	1600	−63	1537	−24	1513	−46	1467	+67	1534
1481	+21	1502	+34	1536	−44	1492	−19	1473	−28	1445	+42	1487
1442	+11	1453	+21	1474	−23	1451	−10	1441	−12	1429	+22	1451
1392	+ 2	1394	+ 7	1401	− 1	1400	+ 1	1401	——	1401	+ 4	1405
1375	−10	1365	− 9	1356	+18	1374	+17	1391	+11	1402	−17	1385
1338	−18	1320	−24	1296	+44	1340	+20	1360	+27	1387	−35	1352
1257	−27	1230	−37	1193	+64	1257	+31	1288	+43	1331	−57	1274
1248	−42	1206	−46	1160	+85	1245	+44	1289	+57	1346	−78	1268

Note.—The extensions are marked +; the compressions are marked —.

Considering the very minute quantities which had to be measured, and the numerous causes of disturbance to which observations of so much delicacy were liable, such as changes of temperature or want of perfect uniformity in the dimensions or texture of the beams, the results, as shown by the column of differences, exhibit more regularity than could have been expected; and they point out the position of the neutral axis, as the centre of the beam, in a manner so decided, as to remove all further doubt upon this subject, not only in the smaller strains, but in the larger ones also; which, in the case of the second beam, were carried to about three-fourths of the breaking weight.

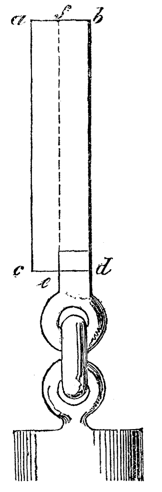
It will be observed also that the extensions and compressions increase in an arithmetical ratio from the centre to the extreme upper and lower sides of the beam.

These experiments having established the fact that the neutral axis is in the centre of a rectangular beam, and that its position is not sensibly altered by variations in the amount of strain applied, it becomes evident that if there were no other elements of strength than the resistances to direct extension and compression, the well-known formula

$$W=\frac{2adf}{3l}$$

should give the breaking weight when f is equal to the smaller of these two resistances, which in cast iron is the tensile resistance. But the weight so calculated is less than half the actual strength of the beam.

In considering this question, I was forcibly struck by the circumstance, that, in applying the law of "*ut tensio sic vis*" to contiguous fibres, under different degrees of tension and compression, the effect of lateral adhesion is omitted, and each fibre is supposed to be capable of taking up the same degree of extension and compression from the same force as if it acted separately, and independently of the adjoining fibres. But it is well known as a practical fact, that there is a powerful lateral action which tends to modify the effect of unequal strains.



If, for example, a bar, *abcd*, have a strain applied at *efdb*, the portion *defb* will not be extended so much as it would be if separated from *acef*, unless an equal strain is applied to the portion *acef*. And if a portion of a bar cannot be extended in proportion to the force applied to it, unless the contiguous part is equally strained, it follows that the outer portions of a beam subjected to transverse strain will not be extended in proportion to the force applied, because the part nearer the neutral axis is not equally strained. The measurements made for obtaining the position of the neutral axis afford direct evidence on this point.

In the first beam, a strain of 5786 lbs. caused an extension of twenty-eight divisions of the micrometer; the points measured were $\frac{11}{12}$ths of the depth of the beam. The extension at the outer fibres was therefore $28\times\frac{12}{11}=30$ divisions. The micrometer reading before the strain was applied was 2111, and the total distance of the points measured was $2111+51661=53772$. The effect of the strain caused therefore an extension of $\frac{30}{53772}=\frac{1}{1792\cdot4}$ of the length. The beam was 7 feet 4 inches long, 6 inches deep, and 2 inches thick; and as

$$W=\frac{2adf}{3l}$$

$$f=\frac{3lW}{2ad}$$

$$\text{or } f=\frac{3\times88\times5786}{2\times12\times6}=10{,}608 \text{ lbs.};$$

so that, with a strain of 10,608 lbs. at the outer fibres, the extension produced was $\frac{1}{1792\cdot4}$ of the length.

But in referring to the experiments made by Mr. Hodgkinson, it will be seen that a force of 10,538, applied by direct tensile strain, extends cast iron $\frac{1}{1056}$th of its length, being nearly double that exhibited by the beam.

In the second beam, a weight of 8000 lbs. (from the mean of two results) produced an extension of forty divisions, which at the extreme fibres will be $40\frac{12}{11}=44$ divisions.

The mean reading of the micrometer, previous to the strain being applied, was 1439; therefore the extension was

$$\frac{44}{51661+1439}=\frac{1}{1207}.$$

The strain at the outer fibres produced by this weight was 14,666 lbs.; so that 14,666 lbs. to the inch caused an extension of $\frac{1}{1207}$th of the length.

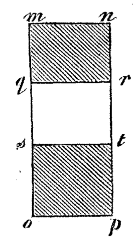
But referring again to Hodgkinson's experiments on direct tensile strain, a weight of 14,793 lbs. produced an extension of $\frac{1}{645}$th of the length; which is again nearly double that produced by the same strain when excited by a weight applied transversely.

From these and other considerations I was led to think it probable that the effect of the lateral action of the fibres or particles of a beam, tending to modify the effect of the unequal strains and opposite forces, and thus diminishing the amount of extension and compression which would otherwise arise, constituted in effect a *resistance to flexure*; and it will be found that the following experiments fully confirm the existence of this resistance as an additional element of strength in beams; and that it explains the apparent anomaly in the amount of tensile resistance when excited by direct and by transverse strains.

Assuming the probability of a resistance, acting independently of, or in addition to, the resistance of direct tension and compression, and varying with the flexure, it occurred to me that it might be exhibited experimentally by casting open girders of the forms shown figs. 2, 3 & 4, having the same sectional area in the upper and lower ribs; the same number of vertical ribs, but the distance between the horizontal ribs, and consequently the deflections of the girders, different.

In these girders the neutral axis would necessarily be (like that of the solid beam) in the centre, and the sectional area of the ribs subjected to tension and compression being the same in each, the circumstances under which rupture would ensue would be similar, except in the amount of flexure.

The formula for the strength of a girder of this form is as follows:—



Let a=the united area contained in the upper and lower ribs;
a'=the intervening space;
d=the total depth;
c=the distance between the upper and lower ribs;
l=the length of bearing;
W=the breaking weight;
and F=the force required to produce rupture in the extreme fibres or particles.

Then $a+a'$=the total area of the rectangle m, n, o, p,

$$W=\frac{2d\mathrm{F}}{3l}(a'+a)-\frac{2ca'}{3l}\times\frac{c\mathrm{F}}{d};$$

or

$$W=\frac{2\mathrm{F}}{3l}\left\{(a'+a)d-\frac{a'c^2}{d}\right\},$$

$$W=\frac{2\mathrm{F}a}{3l}\left(d+c+\frac{c^2}{d}\right).$$

The formula may also be obtained by calculating the moments in the usual way. Using the same letters as before, we have, for the distance of the centres of compression and extension,

$$\frac{2}{3}\left(d+\frac{c^2}{d+c}\right).$$

The force acting when F is the strain which breaks the outer fibre, will be

$$\frac{F+\frac{Fc}{d}}{2}=F\frac{\left(1+\frac{c}{d}\right)}{2}.$$

Hence
$$\frac{W}{2}\times\frac{l}{2}=\frac{2}{3}\left(d+\frac{c^2}{d+c}\right)\left(\frac{1+\frac{c}{d}}{2}\right)\frac{aF}{2};$$

or
$$W=\frac{2Fa}{3l}\left(d+c+\frac{c^2}{d}\right).$$

The value of W being obtained by experiment in each case, we have from the formula

$$F=\frac{3lW}{2a\left(d+c+\frac{c^2}{d}\right)};$$

and if the strength depended only on the direct tensile power of the material, F should in each case be constant, and equal to the direct tensile resistance; but if, in addition to this, there existed another element of strength in the resistance occasioned by the lateral adhesion and varying with the flexure, the value of F would be found, in every case, greater than the tensile resistance, and to increase when the flexure increased.

Four beams were cast of each form, of which the details, the exact dimensions, deflections, and breaking weights are given in the Appendix. The results were as follows, obtained from the mean of four experiments on each form of girder:—

Description of beam.	Total depth of beam.	Sectional area of the two ribs.	Distance between the ribs.	Deflection with nine-tenths of breaking weight.	Breaking weight.
	in.	in.	in.	in.	lbs.
Form No. 2	2·51	1·98	·54	·510	2468
Form No. 3	3·00	2·00	1·00	·401	3119
Form No. 4	4·00	1·98	2·03	·301	4339

The value of F being derived from each of these results by the formula

$$F=\frac{3lW}{2a\left(d+c+\frac{c^2}{d}\right)}.$$

	Deflection.	Value of F.
Form No. 2	·510	35386
Form No. 3	·401	31977
Form No. 4	·301	28032

The tensile strength of the metal obtained from the mean of eight experiments, given in the Appendix, was 18,750 lbs.; here, therefore, was decided evidence, first, that the value of F exceeded the tensile strength in all three forms, and that it increased with the increase of flexure.

In connexion with the above-described experiments, I made four others on solid beams having the same sectional area and length as the open girders; and the mean of the four gave a breaking weight of 1888 lbs. Obtaining the value of F from these experiments, we have,—

Deflection with nine-tenths of breaking weight.	Value of F.
·670	41709 lbs.

which again exhibits an increase in the value of F, with an increase in the deflection.

The foregoing experiments having shown that in girders containing the same depth of metal, the resistance arising from the lateral action of the particles depended on the amount of the flexure, I thought it desirable to make other experiments to ascertain how this resistance varied in girders having the same total depth, and consequently nearly the same deflection, but with different depths of metal in the girder. For this purpose beams were cast of the forms Nos. 5, 6 and 7, each 4 inches deep, and with the upper and lower ribs $1\frac{1}{2}$ inch by $\frac{3}{4}$ inch, the ribs being placed as shown in the figures, so that the depth of the metal in No. 5 was twice as great as in Nos. 6 and 7.

Four beams were cast of each form,—the exact dimensions and breaking weights are given in the Appendix,—and the mean results were as follows:—

Description of beam.	Depth of beam.	Depth of metal.	Sectional area.	Deflection.	Breaking weight.
Form No. 5	4·04	3·01	2·320	·322	5141
Form No. 6	4·04	1·48	2·230	·310	5147
Form No. 7	4·07	1·56	2·380	·262	6000

Obtaining the value of F from these experiments, and comparing them with beam No. 4, which had the same total depth, we have—

	Deflection.	Depth of metal.	Value of F.
Form No. 5	·322	3·01	37408
Form No. 4	·301	1·97	28032
Form No. 7	·262	1·56	27908
Form No. 6	·310	1·48	25271

These experiments did not afford so complete a comparison as the former series, because the intervals between the vertical ribs were not equal, nor in the same proportion to the depth of metal, the effect of which would be to vary to some extent the form of the curve of deflection. Nevertheless, they show in an equally decided manner, that when the deflection is the same the resistance increases when the depth of metal in the beam is increased.

The foregoing experiments have therefore elicited three facts as regards beams formed of two parallel bars separated at given intervals by vertical ribs:—

First, that in every case the resistance, or the value of F, is greater than that due to the tensile resistance of the metal.

Secondly, that with the same depth of metal in the beam, and the same distance of bearing, the resistance is greater when the deflection is greater.

Thirdly, that with the same deflection and the same length of bearing, the resistance is greater when the depth of metal in the beam is greater.

And it follows from these results, that there is an element of strength depending on the amount of deflection in connexion with the depth of metal in the beam, or in other words, dependent upon the degree of flexure to which the metal forming the beam is subjected.

The existence of an element of strength in addition to the resistances to direct tension and compression being clearly proved by these experiments, it becomes interesting to ascertain the law under which it varies, in the form of beams experimented upon.

Now if from the value of F, the tensile strength of the metal is deducted, it will be found that the remainder maintains nearly a constant ratio in each case to the depth of the metal in the beam multiplied by its deflection. It would appear, therefore, that the total resistance, or the value of F, is composed of two quantities; one being constant and limited by the resistance to direct tension, and the other varying directly as the degree of flexure to which the metal forming the beam is subjected.

The applicability of this simple law may be tested by the results of the experiments, as follows:—

Let φ=the resistance to flexure in the solid beam at the time of rupture;
and let D=the depth,
δ=the deflection,
f=tensile resistance,
and F=total resistance.

Then in the solid beam

$$f+\varphi=\mathrm{F};$$

and let F′, D′ and δ', represent the total resistance, depth of metal, and deflection of any other of the beams; then, the lengths being equal, if the resistance arising from the lateral action varies as the depth of metal into the deflection,

$$\mathrm{F}'=f+\varphi\frac{\mathrm{D}'\delta'}{\mathrm{D}\delta}.$$

The value of φ may be determined from this equation, applied to each of the experiments, in two ways; first, by supposing f to be a constant quantity; and secondly, by supposing f and φ to have a constant ratio.

By the first mode, the whole of the errors of observation and irregularities of the strength of the metal would be accumulated in φ. By the second method, these irregularities will be divided between the values of f and φ.

Adopting therefore the second method, let 1 to m represent the ratio of f to φ:

then $$f=m\varphi,$$

and $$m\varphi+\varphi\frac{D'\delta'}{D\delta}=F';$$

or $$\varphi=\frac{F'}{m+\frac{D'\delta'}{D\delta}},$$

which ought to be a constant quantity in all the experiments.

We cannot obtain the deflections at the line of rupture, but they may be assumed to be proportional to the deflections with $\frac{9}{10}$ths of the breaking weights in each case.

Now the value of F in the solid beam was found to be 41,709 lbs.; and the value of f, from the experiments on direct tension, was 18,750 lbs.: and as in the solid beam $$f+\varphi=F,$$ φ will be 22,959 lbs.,

and the ratio of φ to f will be as 1 to ·81.

For the purpose of comparison, I have deduced the value of f and φ, in solid beams, from the experiments of Mr. HODGKINSON on ten different descriptions of metal; the results of which are given in the following Table:—

Description of iron.	Transverse strength of bar 1 inch square and 54 inches between the supports.	Tensile strength per square inch.	Value of $f+\varphi$ from the formula $w=\frac{2ad(f+\varphi)}{3l}$.	Value of φ from the formula $w=\frac{2ad(f+\varphi)}{3l}$, $\varphi=\frac{3lw}{2ad}-f$.
	lbs.	lbs.	lbs.	lbs.
Carron iron No. 2, cold blast	476	16,683	38,556	21,873
Carron iron No. 2, hot blast	463	13,505	37,503	23,998
Carron iron No. 3, cold blast	446	14,200	36,126	21,926
Carron iron No. 3, hot blast...............	527	17,755	42,687	24,932
Devon iron No. 3, hot blast	537	21,907	43,497	21,590
Buffery iron No. 1, cold blast	463	17,466	37,503	20,037
Buffery iron No. 1, hot blast	436	13,434	35,316	21,882
Coed-Talon iron No. 2, cold blast	413	18,855	33,453	14,598
Coed-Talon iron No. 2, hot blast	416	16,676	33,696	17,020
Low Moor iron No. 3, cold blast.........	467	14,535	37,827	23,292
Means	464	16,502	37,616	21,114

The mean ratio of φ to f in these metals appears to be as 1 to ·78. The metal used in my experiments was a mixture consisting of two-thirds of South Staffordshire No. 3, hot blast pig, and one-third old metal recast. As compared with Mr. HODGKINSON'S experiments, its strength accorded nearly with that of the Carron iron No. 3, hot blast.

The mean ratio of φ to f, obtained from Mr. HODGKINSON'S experiments, being as 1 to ·78, and from the experiments herein detailed being as 1 to ·81, we may consider f to be four-fifths of φ; and therefore

$$m=\cdot 8.$$

Using this ratio, the values of φ and f, derived from the formula

$$\varphi=\frac{F}{m+\frac{D'\delta'}{D\delta}}$$

and $$f=\varphi m,$$

as applied to each of the experiments, are given below :—

No. 1. $\varphi=\frac{41709}{\cdot 8+\frac{2\cdot 012\times\cdot 670}{2\cdot 012\times\cdot 670}}=23{,}171$ lbs., $f=18{,}537$ lbs.

No. 2. $\varphi=\frac{35386}{\cdot 8+\frac{1\cdot 97\times\cdot 510}{1\cdot 348}}=22{,}904$ lbs., $f=18{,}323$ lbs.

No. 3. $\varphi=\frac{31977}{\cdot 8+\frac{2\cdot 01\times\cdot 401}{1\cdot 348}}=22{,}890$ lbs., $f=18{,}312$ lbs.

No. 4. $\varphi=\frac{28032}{\cdot 8+\frac{1\cdot 97\times\cdot 301}{1\cdot 348}}=22{,}606$ lbs., $f=18{,}085$ lbs.

No. 5. $\varphi=\frac{37408}{\cdot 8+\frac{3\cdot 01\times\cdot 322}{1\cdot 348}}=24{,}626$ lbs., $f=19{,}501$ lbs.

No. 6. $\varphi=\frac{25270}{\cdot 8+\frac{1\cdot 48\times\cdot 310}{1\cdot 348}}=22{,}167$ lbs., $f=17{,}734$ lbs.

No. 7. $\varphi=\frac{27908}{\cdot 8+\frac{1\cdot 56\times\cdot 262}{1\cdot 348}}=25{,}302$ lbs., $f=20{,}242$ lbs.

These results, though not exhibiting complete regularity, are sufficiently uniform to indicate that the assumed law of the variation of this resistance is a close approximation to the truth. It will be observed also, that Nos. 2, 3, 4 and 6, give a smaller value of φ than Nos. 1, 5 and 7, which probably arises from the difference in the proportion which the distance between the vertical ribs bears to the depth of the metal; a circumstance which would affect, to some extent, the form of the curve of deflection.

In the formula $\varphi=\frac{F'}{m+\frac{D'\delta'}{D\delta}}$, $\frac{D'\delta'}{D\delta}$ represents the ratio of the depth of metal in each beam multiplied by its deflection, to the depth of metal in the solid beam multiplied by its deflection. But the deflections, as might have been expected from known laws, were nearly in the inverse ratio of the total depths of each girder; therefore the degree of flexure, and consequently the resistance to flexure in each, will be nearly as the depth of metal divided by the total depth of the girder, and we are thus enabled to obtain a formula for computing, approximately, the breaking weights of these girders, without first ascertaining their deflection.

Using the same letters as before, we have, for the resistance due to tension,

$$\frac{2a}{3l}\left(d+c+\frac{c^2}{d}\right)f;$$

and for the resistance to flexure,

$$\frac{2a}{3l}\left(d+c+\frac{c^2}{d}\right)\frac{\phi D}{d};$$

and consequently, for the united effect of the two resistances,

$$W=\frac{2a}{3l}\left(d+c+\frac{c^2}{d}\right)\left(f+\frac{\phi D}{d}\right).$$

I shall therefore conclude these observations by comparing the breaking weights computed for tensile resistance alone, and those obtained from the formula which includes the resistance to flexure, with the actual breaking weights obtained by the experiments, taking the value of $f=18{,}750$ lbs., and $\phi=23{,}000$ lbs.

Description of beam or girder.	Breaking weight if the resistance depended on direct tensile strength.	Breaking weight computed by the formula, including the resistance to flexure.	Breaking weight as obtained by the experiments.
	lbs.	lbs.	lbs.
No. 1	849	1890	1888
No. 2	1308	2567	2468
No. 3	1808	3287	3084
No. 4	2912	4659	4353
No. 5	2578	4935	5141
No. 6	3819	5533	5147
No. 7	4031	5919	6000

The accordance exhibited by the computed and the actual breaking weights, evinces the general accuracy of the formula, as applied to this form of beam ; while these results, compared with those computed for direct tensile force alone, show how large a proportion of the strength of cast iron, when subjected to transverse strain, is due to the resistance arising from the lateral action.

It will also be seen that comparisons of the relative strengths of different forms of section, calculated, as has been customary, on the assumption that the resistances are constant forces, or governed by a constant coefficient, must be entirely fallacious.

It was my intention to have included in this paper a similar investigation as to the position of the neutral axis, and the amount of the resistance arising from lateral action of the fibres in wrought iron; but as the experiments will take some time to complete, and as the facts elicited in reference to cast iron are of sufficient importance to render it desirable that they should be made known, I will reserve the examination of wrought iron for the subject of another communication.

Girder No. 1.

	Experiment No. 1.	Experiment No. 2.	Experiment No. 3.	Experiment No. 4.
	inches.	inches.	inches.	inches.
Depth	2·015	2·02	2·073	2·040
Thickness	·975	·98	1·030	·990
Area of section	1·965	1·98	2·135	2·020
Weight applied, lbs.	Deflection.	Deflection.	Deflection.	Deflection.
40	·015	·013	·014	·014
376	·145	·115	——	——
600	·203	——	——	——
712	·280	·233	·264	·244
936	·330	——	——	——
1160	·490	·420	·397	·414
1608	·725	·625	·579	·614
1664	Broke·755	·655	——	——
1720	......	·680	·629	·659
1832	......	·737	·679	·734
1888	......	Broke	·699	·764
1916	......	......	——	Broke
1944	......	......	·734	
2000	......	......	·762	
2028	......	......	·774	
2056	......	......	·789	
2084	......	......	Broke	
Breaking weight, lbs.	1664	1888	2084	1916
Deflection with nine-tenths of breaking weight, inches	·643	·667	·699	·670

Girder No. 2.

	Experiment No. 1.	Experiment No. 2.	Experiment No. 3.	Experiment No. 4.
	inches.	inches.	inches.	inches.
Total depth	2·54	2·53	2·49	2·50
Depth between upper and lower ribs	·56	·55	·51	·55
Area of top rib	1·00	1·00	·97	·98
Area of bottom rib	1·01	1·00	·99	·97
Weight applied, lbs.	Deflection.	Deflection.	Deflection.	Deflection.
40	·009	·007	·007	·007
712	——	·132	·134	·137
804	·199	——	——	——
1292	·304	——	——	——
1516	——	·302	·319	·312
1740	·414	——	——	——
1852	——	·372	——	——
1964	·489	·397	·426	·433
2076	——	·427	——	——
2188	Broke	·445	·479	·487
2300	......	·479	·526	·532
2412	......	·512	Broke	·550
2524	......	·542	......	Broke
2636	......	·575		
2748	......	Broke		
Breaking weight, lbs.	2188	2748	2412	2524
Deflection with nine-tenths of breaking weight, inches	·489	·532	·482	·516

Girder No. 3.

	Experiment No. 1.	Experiment No. 2.	Experiment No. 3.	Experiment No. 4.
	inches.	inches.	inches.	inches.
Total depth	3·02	3·00	3·00	3·00
Depth between upper and lower ribs	·98	1·00	1·01	1·01
Area of top rib	1·03	1·02	·97	1·01
Area of bottom rib	·99	·98	1·01	·97
Weight applied, lbs.	Deflection.	Deflection.	Deflection.	Deflection.
40	·006	·005	·005	·005
712	——	·085	·085	·085
844	·113	——	——	——
1516	·216	·185	·197	·195
1740	·248	——	——	——
2188	·328	——	·297	·293
2300	——	·295	——	——
2524	·388	——	——	——
2636	·418	——	·363	·375
2748	·433	·377	——	——
2972	·483	·410	·423	Broke
3028	Broke	·425	·438	
3084	......	·435	·452	
3112	......	·437	Broke	
3224	......	Broke		
Breaking weight, lbs.	3028	3224	3112	2972
Deflection with nine-tenths of breaking weight, inches	·435	·402	·397	·371

Girder No. 4.

	Experiment No. 1.	Experiment No. 2.	Experiment No. 3.	Experiment No. 4.
	inches.	inches.	inches.	inches.
Total depth	3·99	4·00	3·99	4·01
Depth between upper and lower ribs	2·00	2·03	2·05	2·04
Area of top rib	1·00	·97	·98	·98
Area of bottom rib	1·00	·99	·98	1·01
Weight applied, lbs.	Deflection.	Deflection.	Deflection.	Deflection.
40	·002	·002	·002	·003
712	·047	·040	·048	·058
1516	·104	·097	·102	·108
1964	·134	——	——	——
2188	·161	·155	·155	·148
2636	·199	·197	·185	·183
3084	·227	·227	·223	·218
3420	——	·259	——	——
3532	·269	·267	·255	·253
3756	·299	·282	·285	——
3980	·317	·312	·300	·303
4092	·329	·320	·307	——
4148	·336	·322	·313	——
4204	Broke	·327	Broke	——
4260	......	Broke	......	·333
4316	......	......	......	——
4400	......	......	.....	·343
4428	......	......	......	——
4745	......	......	......	Broke
Breaking weight, lbs.	4204	4260	4204	4745
Deflection with nine-tenths of breaking weight, inches	·297	·293	·282	·331

Girder No. 5.

	Experiment No. 1.	Experiment No. 2.	Experiment No. 3.	Experiment No. 4.
	inches.	inches.	inches.	inches.
Total depth	4·02	4·05	4·05	4·04
Depth between upper and lower ribs	1·04	1·04	1·04	1·00
Area of top rib	1·125	1·16	1·14	1·22
Area of bottom rib..............	1·162	1·13	1·15	1·20
Weight applied, lbs.	Deflection.	Deflection.	Deflection.	Deflection.
712	Deflections uncertain from imperfect fastening of the cord.	——	——	——
1516		——	——	——
2188		——	——	——
2290		·133	·148	·138
2636		——	——	——
2885		·173	·182	·178
3084		——	——	——
3445		·213	·221	·223
3532		——	——	——
3980		——	——	——
4005		·268	·270	·259
4428		——	——	——
4565		·313	·320	·308
4652		——	——	——
4705		·323	·335	——
4845		·348	·350	·335
4876		——	——	——
4927		——	——	——
4985		·348	Broke	·340
5008		——	......	——
5050	Broke	——	......	——
5125	......	Broke	......	·355
5265	......	......	......	·365
5405	......	......	......	Broke
Breaking weight, lbs............	......	5125	4985	5405
Deflection with nine-tenths of breaking weight, inches	......	·321	·313	·331

Girder No. 6.

	Experiment No. 1.	Experiment No. 2.	Experiment No. 3.	Experiment No. 4.
	inches.	inches.	inches.	inches.
Total depth	4·02	4·05	4·03	4·06
Depth between upper and lower ribs	2·52	2·55	2·56	2·61
Area of top rib	1·13	1·18	1·08	1·10
Area of bottom rib	1·13	1·09	1·11	1·10
Weight applied, lbs.	Deflection.	Deflection.	Deflection.	Deflection.
712	Deflections uncertain from the same cause as Experiment 1, No. 5.	—	—	—
1516		—	—	—
2188		—	—	—
2290		·130	·138	·138
2636		—	—	—
2885		·168	·186	·175
3084		—	—	—
3445		·205	·220	·222
3532		—	—	—
3980		—	—	—
4005		·251	·263	·272
4428		—	—	—
4565		·300	·313	·313
4652		—	—	—
4845		·315	Broke	·350
4876		—	......	—
4988		—	......	·365
5100		—	......	—
5125	......	Broke	......	·378
5212	Broke	......	......	—
5265	......	......	......	·382
5405	......	......	......	Broke
Breaking weight, lbs.	......	5125	4845	5405
Deflection with nine-tenths of breaking weight, inches	......	·298	·293	·340

Girder No. 7.

	Experiment No. 1.	Experiment No. 2.	Experiment No. 3.	Experiment No. 4.
	inches.	inches.	inches.	inches.
Total depth	4·05	4·10	4·08	4·05
Depth between upper and lower ribs	2·50	2·51	2·51	2·52
Area of top rib	1·19	1·26	1·21	1·16
Area of bottom rib	1·19	1·19	1·17	1·16
Weight applied, lbs.	Deflection.	Deflection.	Deflection.	Deflection.
2290	·105	·105	·095	·090
2885	·115	·130	·120	·125
3445	·150	·160	·140	·160
4005	·185	·185	·180	·182
4565	·217	·215	·215	·210
5125	·255	·250	·235	·237
5405	·272	·267	—	—
5685	Broke	·285	·270	·272
5825	......	·292	—	Broke
5965	......	·305	Broke	
6105	......	·310		
6245	......	·320		
6385	......	·330		
6525	......	Broke		
Breaking weight, lbs.	5685	6525	5965	5825
Deflection with nine-tenths of breaking weight, inches	·252	·297	·253	·246

Summary of the Experiments on Transverse Strength, giving the mean results.

	Depth.	Sectional area.	Distance between the ribs.	Breaking weight.	Deflection with nine-tenths of breaking weight.
	in.	sq. in.	in.	lbs.	in.
Form of beam No. 1.	2·015 2·020 2·073 2·040	1·965 1·980 2·135 2·020	....	1664 1888 2084 1916	·643 ·667 ·699 ·670
Mean	2·012	2·025	......	1888	·670
Form of beam No. 2.	2·54 2·53 2·49 2·50	2·01 2·00 1·96 1·95	·56 ·55 ·51 ·55	2188 2748 2412 2524	·489 ·532 ·482 ·516
Mean	2·51	1·98	·54	2468	·510
Form of beam No. 3.	3·02 3·00 3·00 3·00	2·02 2·00 1·98 1·98	·98 1·00 1·01 1·01	3028 3224 3112 2972	·435 ·402 ·397 ·371
Mean	3·01	2·00	1·00	3084	·401
Form of beam No. 4.	3·99 4·00 3·99 4·01	2·00 1·96 1·96 1·99	2·00 2·03 2·05 2·04	4204 4260 4204 4745	·297 ·293 ·282 ·331
Mean	4·00	1·98	2·03	4353	·301
Form of beam No. 5.	4·02 4·05 4·05 4·04	2·287 2·290 2·290 2·420	1·04 1·04 1·04 1·00	5050 5125 4985 5405	...... ·321 ·313 ·331
Mean	4·04	2·322	1·03	5141	·322
Form of beam No. 6.	4·02 4·05 4·03 4·06	2·26 2·27 2·19 2·20	2·52 2·55 2·56 2·61	5212 5125 4845 5405	...... ·298 ·293 ·340
Mean	4·04	2·23	2·56	5147	·310
Form of beam No. 7.	4·05 4·10 4·08 4·05	2·38 2·45 2·38 2·32	2·50 2·51 2·51 2·52	5685 6525 5965 5825	·252 ·297 ·253 ·246
Mean	4·07	2·38	2·51	6000	·262

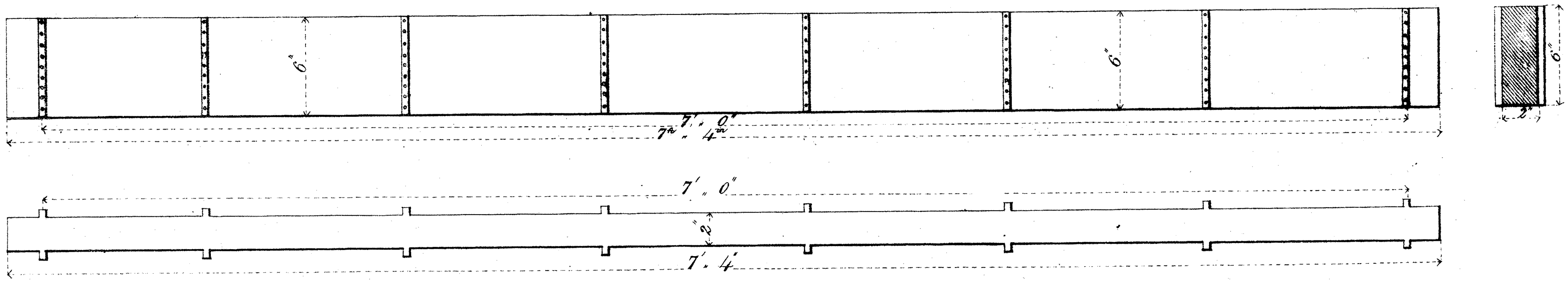
Experiments on Direct Tension.

Number of experiment.	Sectional area at the place of fracture.	Last weight supported.	Weight with which the bar broke.	Remarks.
	inches.	lbs.	lbs.	
1.	1·0506	18,560	18,840	A small air-bubble.
2.	1·0557	19,680	19,960	A small air-bubble.
3.	1·0100	21,360	21,500	A small air-bubble at corner, very small.
4.	1·0364	16,320	16,320	Honey-combed.
5.	1·0301	17,440	17,440	Sound.
6.	1·0403	16,320	17,440	A small air-bubble.
7.	1·0150	21,640	21,920	Sound.
8.	1·0200	22,200	22,470	Sound.
Mean	1·0323	19,190	19,486	

Mean greatest weight supported, per inch . . . . 18,590 lbs.
Mean weight which broke the bar, per inch . . . 18,876 lbs.

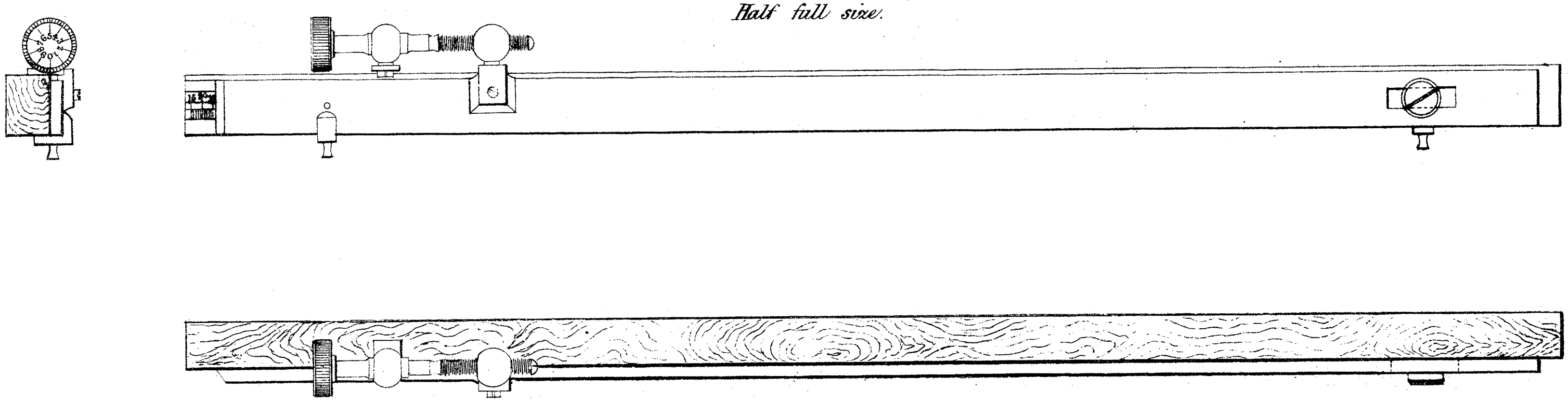
Considering the actual breaking weight to be between these two, and rather nearer the latter, when due allowance is made for the small air-bubbles, the mean breaking weight may be taken at 18,750 lbs. per square inch.

Beam employed in determining the position of the neutral axis.

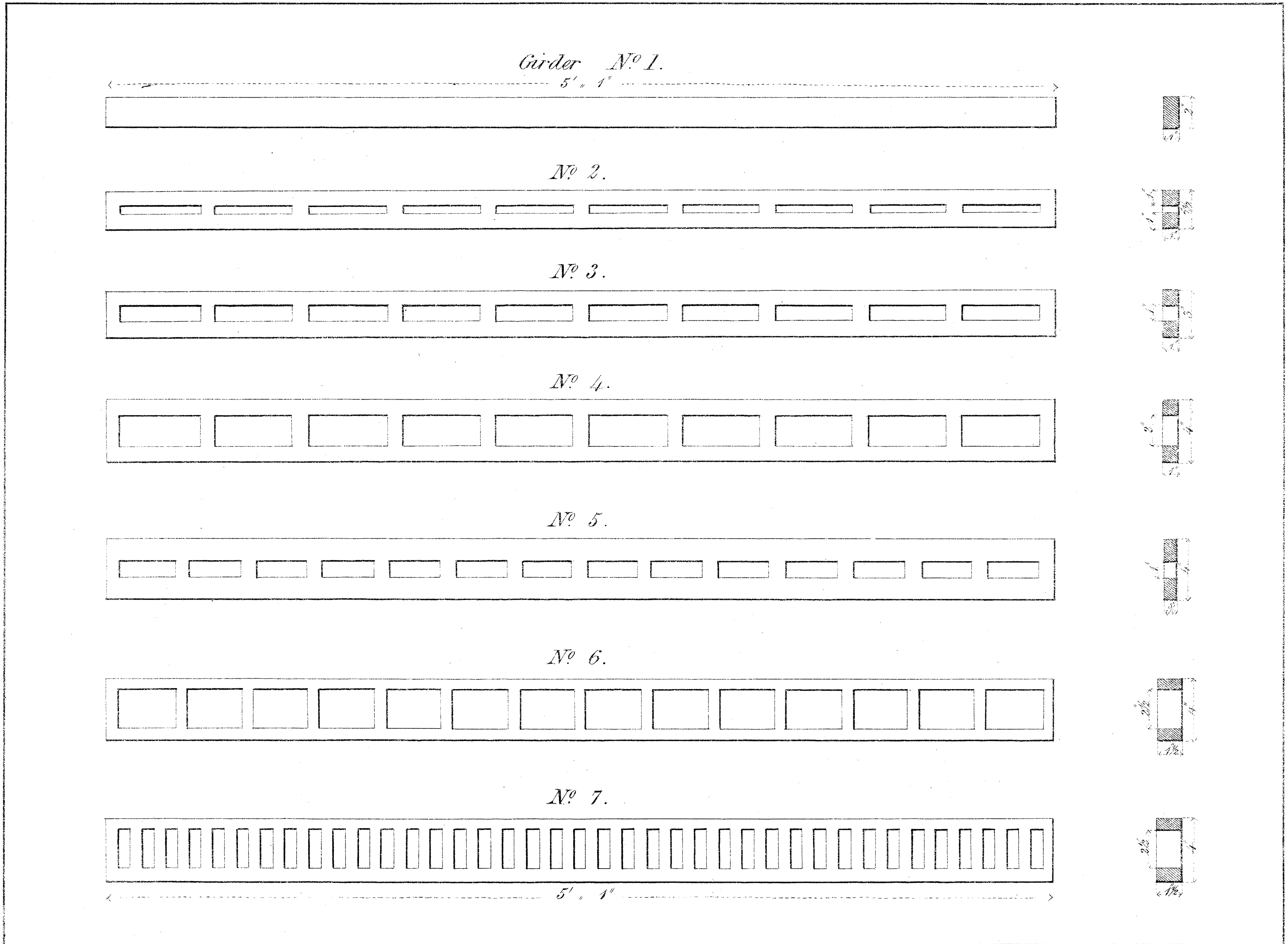


Measuring Instrument.

Half full size.



J. Basire sc.



J. Basire Sc.